17. A. A. Il'yushin, Plasticity: Principles of the General Mathematical Theory [in Russian], Izd. Akad. Nauk SSSR, Moscow (1963).
18. A. A. Movchan, A Hypothesis on Damage Accumulation in Plastic Strain [in Russian], MAI, Moscow (1982). Manuscript deposited with VINITI, No. 3797-81 Dep.
19. A. A. Movchan, "The damage-summation law in complex modes of plastic strain," Probl. Prochn., No. 8 (1981).
20. V. Weiss, J. Sessler, and P. Lackman, "Effect of several parameters on low cycle fatigue behavior," Acta Metallurgica, 11, No. 4 (1963).
21. D. E. Martin, "An energy criterion for low-cycle fatigue," Trans. ASME, Ser. D, J. Basic Eng., 83, No. 4 (1961).
22. D. C. Drucker, G. Mylonas, and G. Lianis, "Exhaustion of ductility of E-steel following compressive prestrain," Welding J., 39, No. 3 (1960).

FORMULATION OF A DIAGNOSIS PROBLEM FOR A THERMOELASTIC MEDIUM

V. A. Lomazov and Yu. V. Nemirovskii

UDC 539.3

By diagnosis problem we understand the determination of the characteristics of a medium from information obtained from a certain number of tests (test studies). Similar formulations are widely used in geophysics, particularly in seismic surveying. The general methods available have been discussed in [1]. Typical applications of these formulations and methods in diagnosis as regards the mechanics of deformable solids are related to the identification of unsatisfactory items, determining wear during use, and researching the effects of external factors on the properties of materials.

Here we deal with the determination of small changes in the thermoelastic characteristics of a material whose original properties are known. This can be interpreted as refining the properties of the material. In fact, when an item is manufactured, the material is subject to external factors arising from the production technology, which in general alter its properties. A method is proposed for determining the new thermoelastic characteristics on the assumption that these remain close to those of the medium that was originally homogeneous and isotropic. We consider an example of using this method.

1. The propagation of thermoelastic waves in an inhomogeneous anisotropic medium is described [2] by the following equations:

$$\rho \ddot{u}_i = (C_{ijkl} u_{k,l})_{,j} - (\beta_{ij}\Theta)_{,j}; \tag{1.1}$$

$$C_\varepsilon \dot{\Theta} - (K_{ij}\Theta_{,i})_{,j} = 0, \tag{1.2}$$

where ρ is density, Θ is relative temperature, $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector, $\beta_{ij} = C_{ijkl}\alpha_{kl}$; α_{kl} are the thermal-expansion coefficient C_{ijkl} are the isothermal rigidity coefficients, and K_{ij} are the thermal conductivities. All of these quantities are functions of the spatial variables $\mathbf{x} = (x_1, x_2, x_3)$, $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$, $\Theta = \Theta(\mathbf{x}, t)$.

We denote by ρ^0, C^0_{ijkl}, β^0_{ij}, C^0_ε, K^0_{ij} the quantities characterizing the thermoelastic properties of a homogeneous isotropic medium. In that case, these quantities are constants, and the tensors C^0_{ijkl}, β^0_{ij}, K^0_{ij} have a specific (simpler) form [2].

In what follows we assume that the medium is weakly inhomogeneous and weakly anisotropic, i.e., the quantities $|\rho - \rho^0|$, $|C_\varepsilon - C^0_\varepsilon|$, $|C_{ijkl} - C^0_{ijkl}|$, $|\beta_{ij} - \beta^0_{ij}|$, $|K_{ij} - K^0_{ij}|$ have identical small orders

Krasnoyarsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 131-137, September-October, 1984. Original article submitted July 11, 1983.

$O(\varepsilon)$, $\varepsilon \ll 1$, whereas the quantities ρ^0, C^0_ε, C^0_{ijkl}, β^0_{ij}, K^0_{ij} are of order $O(1)$. The concept of a slightly inhomogeneous slightly anisotropic medium is natural, since many natural and artificial materials are close in properties to homogeneous and isotropic ones. On the other hand, there may be minor changes in properties in a material initially homogeneous and isotropic produced by quite weak external factors, such as irradiation, and these may lead to substantial changes in behavior. In that case, the diagnostic problem can be treated on the above basis as research on the changes in properties produced by the external factors, and also on determining the character and intensity of those factors from the changes in properties, with the specimen acting as an indicator.

We denote by $u^0_i(x, t)$, $\theta^0(x, t)$ the solution to the equations

$$\rho^0 \ddot{u}^0_i = C^0_{ijkl} u^0_{k,lj} - \beta^0_{ij}\theta^0_{,j}, \quad i, j, k, l = \overline{1,3}; \tag{1.3}$$

$$C^0_\varepsilon \dot{\theta}^0 - K^0_{ij}\theta^0_{,ij} = 0. \tag{1.4}$$

In what follows we assume that the functions $u^\varepsilon_i = u_i(x, t) - u^0_i(x, t)$, $\theta^\varepsilon = \theta(x, t) - \theta^0(x, t)$ and their first and second derivatives with respect to x_i, $i = \overline{1, 3}$ are of order of smallness $O(\varepsilon)$, $\varepsilon \ll 1$. Here it is assumed that $u_i(x, t)$, $\theta(x, t)$ and $u^0_i(x, t)$, $\theta^0(x, t)$ satisfy the same initial and boundary conditions correspondingly. In that case we neglect terms of order $O(\varepsilon^2)$ and take the functions $u_i(x, t)$, $\theta(x, t)$ as bounded together with their partial derivatives with respect to x_i and t up to the second order inclusive, and then (1.1) and (1.2) on the basis of (1.3) and (1.4) reduce to

$$\rho^0 \ddot{u}^\varepsilon_i - C^0_{ijkl} u^\varepsilon_{k,lj} + \beta^0_{ij}\theta^\varepsilon_{,j} = f_i; \tag{1.5}$$

$$C^0_\varepsilon \dot{\theta}^\varepsilon - K^0_{ij}\theta^\varepsilon_{,ij} = g, \text{ where} \tag{1.6}$$

$$f_i = -\rho^\varepsilon \ddot{u}^0_i + \left(C^\varepsilon_{ijkl} u^0_{k,l}\right)_{,j} - \left(\beta^\varepsilon_{ij}\theta^0\right)_{,j}; \tag{1.7}$$

$$g = -C^\varepsilon_\varepsilon \dot{\theta}^0 + \left(K^\varepsilon_{ij}\theta^0_{,i}\right)_{,j}, \quad \rho^\varepsilon = \rho - \rho^0, \quad C^\varepsilon_{ijkl} = C_{ijkl} - C^0_{ijkl}, \quad \beta^\varepsilon_{ij} = \beta_{ij} - \beta^0_{ij},$$
$$C^\varepsilon_\varepsilon = C_\varepsilon - C^0_\varepsilon, \quad K^\varepsilon_{ij} = K_{ij} - K^0_{ij}, \quad i, j, k, l = \overline{1, 3} \tag{1.8}$$

We use these equations to solve the diagnostic problem for a slightly inhomogeneous slightly anisotropic thermoelastic medium. The problem is treated as follows.

Let equations (1.1) and (1.2) apply in the region $\infty < x_1, x_2 < \infty$, $0 \leqslant x_3 < \infty$, $t_0 \leqslant t \leqslant \infty$; we consider m different boundary-value problems for these equations. The solution to boundary value n is denoted by $u^{(n)}_i$, $\theta^{(n)}$. Let this correspond to the initial conditions

$$u^{(n)}_i(\mathrm{x}, t_0) = \varphi^{(n)}_i(\mathrm{x}), \quad n = \overline{1, m}, \; i = \overline{1, 3}; \tag{1.9}$$

$$\dot{u}^{(n)}_i(\mathrm{x}, t_0) = 0; \tag{1.10}$$

$$\theta^{(n)}(\mathbf{x}, t_0) = \psi^{(n)}(\mathbf{x}) \tag{1.11}$$

and the boundary conditions

$$\frac{\partial}{\partial x_3} u^{(n)}_i(x_1, x_2, 0, t) = 0; \tag{1.12}$$

$$\frac{\partial}{\partial x_3} \theta^{(n)}(x_1, x_2, 0, t) = 0. \tag{1.13}$$

Because of the symmetry, the $C_{ijkl}(x)$ tensor contains 21 independent components, while the tensor $\beta_{ij}(x)$ contains six, and the tensor $K_{ij}(x)$ also six [2]. Then the task in the general case consists in determining 33 functions of the spatial variables by reference to additional information derived from the tests.

We assume that we have some information on the solution to the m boundary-value problems which takes the form

$$\operatorname{div} \mathbf{u}^{(n)}(x_1, x_2, 0, t) = \chi_1^{(n)}(x_1, x_2, t); \tag{1.14}$$

$$\operatorname{rot} \mathbf{u}^{(n)}(x_1, x_2, 0, t) = \chi_2^{(n)}(x_1, x_2, t); \tag{1.15}$$

$$\Theta^{(n)}(x_1, x_2, 0, t) = \chi_3^{(n)}(x_1, x_2, t), \quad (x_1^2 + x_2^2)^{1/2} < r, \quad r > 0, \quad n = \overline{1, m}. \tag{1.16}$$

We note that it follows directly from (1.15) that the vector function $\chi_2^{(n)} = (\chi_{21}^{(n)}, \chi_{22}^{(n)}, \chi_{23}^{(n)})$ cannot be arbitrary. The condition $\operatorname{div} \chi_2^{(n)} = 0$ should be obeyed by this.

We assume also that ρ°, C°_{ijkl}, β°_{ij}, K°_{ij}, C°_ε are known, and therefore $u_i^{\circ(n)}$, $\Theta^{\circ(n)}$ may also be considered as known, as these satisfy the thermoelastic equations containing these coefficients and the initial and boundary conditions coincident with (1.9)-(1.13). Then the task in fact amounts to determining C^ε_{ijkl}, β^ε_{ij}, K^ε_{ij}.

2. We now examine this task. We have shown above that (1.1) and (1.2) reduce to (1.5)-(1.8) on the basis of the above assumptions. We first determine the right sides $f_i^{(n)}$, $g^{(n)}$ in the equations of the form of (1.5) and (1.6) corresponding to test n, i.e., boundary-value problem n with the initial and boundary conditions of (1.9)-(1.13). We apply the div and rot operators to these equations to get

$$\rho^0 \ddot{v}^{(n)} - (\lambda^0 + 2\mu^0)\Delta v^{(n)} + \beta^0 \Delta T^{(n)} = F_1^{(n)}; \tag{2.1}$$

$$\rho^0 \ddot{\boldsymbol{\omega}}^{(n)} - \mu^0 \Delta \boldsymbol{\omega}^{(n)} = \mathbf{F}_2^{(n)}; \tag{2.2}$$

$$C_\varepsilon^0 \dot{T}^{(n)} - K^0 \Delta T^{(n)} = F_3^{(n)}, \tag{2.3}$$

where $v^{(n)} = \operatorname{div} \mathbf{u}^{\varepsilon(n)}$, $\boldsymbol{\omega}^{(n)} = \operatorname{rot} \mathbf{u}^{\varepsilon(n)}$, $T^{(n)} = \Theta^{\varepsilon(n)}$, $F_1^{(n)} = \operatorname{div} \mathbf{f}^{(n)}$, $\mathbf{F}_2^{(n)} = \operatorname{rot} \mathbf{f}^{(n)}$, $F_3^{(n)} = g^{(n)}$, λ^0, μ^0 are Lamé constants. These completely characterize the elastic properties of a homogeneous isotropic medium [2]. Here the elements in the rigidity tensor are related to the Lamé constants by: $C^0_{1111} = C^0_{2222} = C^0_{3333} = \lambda^0 + 2\mu^0$, $C^0_{1122} = C^0_{1133} = C^0_{2233} = \lambda^0$, $C^0_{1112} = C^0_{1113} = C^0_{1123} = C^0_{1213} = C^0_{2213} = C^0_{1323} = C^0_{2212} = C^0_{2223} = C^0_{1223} = C^0_{3313} = C^0_{3312} = C^0_{3323} = 0$, $C^0_{1212} = C^0_{1313} = C^0_{2323} = 2\mu^0$, $\beta^0_{ij} = \delta_{ij}\beta^0$, β^0 being the bulk thermal-expansion coefficient, $K^\circ_{ij} = \delta_{ij}K^\circ$, K° the thermal conductivity, and $i, j = \overline{1, 3}$. All these quantities are characteristics of a homogeneous isotropic medium.

We note that if the initial conditions of (1.9)-(1.11) are chosen such that they satisfy homogeneous static equations for thermoelastic equilibrium with constant coefficients C°_{ijkl}, β°_{ij}, K°_{ij}, then $u_i^{\circ(n)}$, $\Theta^{\circ(n)}$ are independent of time t, so $F_1^{(n)} = F_1^{(n)}(\mathbf{x})$, $\mathbf{F}_2^{(n)} = \mathbf{F}_2^{(n)}(\mathbf{x})$, $F_3^{(n)} = F_3^{(n)}(\mathbf{x})$, $n = \overline{1, m}$. We thus have the task of finding the $F_1^{(n)}$, $F_2^{(n)}$, $F_3^{(n)}$ appearing in (2.1)-(2.3) with the initial conditions

$$v^{(n)}(\mathbf{x}, t_0) = 0, \quad \dot{v}^{(n)}(\mathbf{x}, t_0) = 0; \tag{2.4}$$

$$\boldsymbol{\omega}^{(n)}(\mathbf{x}, t_0) = 0, \quad \dot{\boldsymbol{\omega}}^{(n)}(\mathbf{x}, t_0) = 0; \tag{2.5}$$

$$T^{(n)}(\mathbf{x}, t_0) = 0 \tag{2.6}$$

and the boundary conditions

$$\frac{\partial}{\partial x_3} v^{(n)}(x_1, x_2, 0, t) = 0; \tag{2.7}$$

$$\frac{\partial}{\partial x_3} \boldsymbol{\omega}^{(n)}(x_1, x_2, 0, t) = 0; \tag{2.8}$$

$$\frac{\partial}{\partial x_3} T^{(n)}(x_1, x_2, 0, t) = 0. \tag{2.9}$$

The unknown right sides of (2.1)-(2.3) are found from additional information:

$$v^{(n)}(x_1, x_2, 0, t) = \chi_1^{(n)}(x_1, x_2, t) - \operatorname{div} \mathbf{u}^{0(n)}(x_1, x_2, 0, t); \tag{2.10}$$

$$\omega^{(n)}(x_1, x_2, 0, t) = \chi_2^{(n)}(x_1, x_2, t) - \operatorname{rot} \mathbf{u}^{0(n)}(x_1, x_2, 0, t); \tag{2.11}$$

$$T^{(n)}(x_1, x_2, 0, t) = \chi_3^{(n)}(x_1, x_2, t) - \Theta^{0(n)}(x_1, x_2, 0, t). \tag{2.12}$$

The additional information of (2.10)-(2.12) is defined in the following region, as is the previous information of (1.14)-(1.16):

$$(x_1^2 + x_2^2)^{1/2} < r, \ r > 0; \ t_0 \leqslant t < \infty$$

We note that this problem in turn splits up into three types of inverse problem requiring sequential solution.

Problem I. Determination of $F_2^{(n)}$ in (2.2), (2.5), and (2.8) from the information of (2.11).

Problem II. Determination of $F_3^{(n)}$ in (2.3), (2.6), and (2.9) from the information of (2.12).

Problem III. Determination of $F_1^{(n)}$ in (2.1), (2.4), and (2.7) from the information of (2.10) and the use of the solution to problem II.

3. We first consider the auxiliary problem IV. We determine the unknown function $\Phi(\mathbf{x})$ $-\infty < x_1, x_2 < \infty, 0 \leqslant x_3 < \infty$ from the information

$$U(x_1, x_2, 0, t) = \chi(x_1, x_2, t), \quad (x_1^2 + x_2^2)^{1/2} < r, \quad r > 0 \tag{3.1}$$

on the solution U(x, t) to the problem

$$\ddot{U} - \Delta U = \Phi; \tag{3.2}$$

$$U(\mathbf{x}, t_0) = 0, \quad \dot{U}(\mathbf{x}, t_0) = 0; \tag{3.3}$$

$$\frac{\partial}{\partial x_3} U(x_1, x_2, 0, t) = 0. \tag{3.4}$$

We differentiate (3.1), (3.2), and (3.4) partially with respect to t and put $\dot{U}(\mathbf{x}, t)$ – $V(\mathbf{x}, t)$; then as regards V we have

$$\ddot{V} - \Delta V = 0; \tag{3.5}$$

$$V(\mathbf{x}, t_0) = 0; \tag{3.6}$$

$$\frac{\partial}{\partial x_3} V(x_1, x_2, 0, t) = 0; \tag{3.7}$$

$$V(x_1, x_2, 0, t) = \dot{\chi}(x_1, x_2, t), \quad (x_1^2 + x_2^2)^{1/2} < r \quad r > 0. \tag{3.8}$$

A problem of the type of (3.5)-(3.8) was considered in [3], where an explicit expression for $\dot{V}(\mathbf{x}, t_0)$ was obtained. However $\dot{V} = \ddot{U}$. We therefore substitute this expression into (3.2) and use (3.3) and the fact that $\Phi = \Phi(\mathbf{x})$ to get $\Phi(\mathbf{x}) = \dot{V}(\mathbf{x}, t_0)$, which enables us to use the solution of [3] to get an explicit expression for $\Phi(\mathbf{x})$ at $r = \infty$:

$$\Phi\left(x_1, x_2, \left(\tau - x_1^2 - x_2^2\right)^{1/2}\right) = \frac{\left(\tau - x_1^2 - x_2^2\right)^{1/2}}{4\pi} \times$$
$$\times \int\limits_{|\xi|=1} dS_{\xi} \left(\frac{2J(\xi, \tau^{1/2}, \tau)}{\tau^{1/2} - x_1\xi_1 - x_2\xi_2} - \int\limits_{-\tau^{1/2}}^{\tau^{1/2}} \frac{\frac{\partial}{\partial \eta} J(\xi, \eta, \tau)}{\eta - x_1\xi_1 - x_2\xi_2} d\eta \right), \tag{3.9}$$
$$J(\xi, \eta, \tau) = -\frac{2}{\pi^2} \int\limits_{-\infty}^{\infty} dp \int\limits_{0}^{\infty} \frac{\dot{\chi}(\xi_1 p, \ \xi_2 p, \ t)}{t^2 - p^2 + 2p\eta - \tau} dt,$$

$$x_3=(\tau-x_1^2-x_2^2)^{1/2},\quad \xi=(\xi_1,\xi_2),\quad \xi_1^2+\xi_2^2=1,\quad \tau>\eta^2.$$

4. We now solve problem I. A natural change of variables after the corresponding change in symbols reduces (2.2) to dimensionless form. We have

$$\ddot{\omega}_i^{(n)}-\Delta\omega_i^{(n)}=F_{2i}^{(n)},\quad i=\overline{1,3},\quad n=\overline{1,m},$$
$$\omega_i^{(n)}(\mathbf{x},t_0)=0,\quad \dot{\omega}_i^{(n)}(\mathbf{x},t_0)=0,$$
$$\frac{\partial}{\partial x_3}\omega_i^{(n)}(x_1,x_2,0,t)=0,$$
$$\omega_i^{(n)}(x_1,x_2,0,t)=\chi_{2i}^{(n)}(x_1,x_2,t)-(\mathrm{rot})_i\mathbf{u}^{0(n)}(x_1,x_2,0,t),$$
$$(x_1^2+x_2^2)^{1/2}<r,\quad r>0.$$

We note that problem I splits up into 3m independent problems of the type of IV. The determination of $F_{21}^{(n)}$ amounts to substituting the expressions $\left(\dot{\chi}_{2i}^{(n)}(x_1,x_2,t)-(\mathrm{rot})_i\dot{\mathbf{u}}^{0(n)}(x_1,x_2,0,t)\right)$ instead of $\dot{\chi}(x_1,x_2,t)$ into (3.9). Then the dimensional functions are restored by the reverse substitution.

5. We now solve problem II. We reduce (2.3) to dimensionless form. There is a relationship [4] between the solutions to the Cauchy problem for the thermal-conduction equation and for the wave equation with an appropriate correspondence between the initial conditions. This relationship enables us in our case to get

$$T^{(n)}(\mathbf{x},t)=\frac{1}{\sqrt{\pi t}}\int_0^\infty \exp(-\tau^2/4t)\,W^{(n)}(\mathbf{x},\tau)\,d\tau,\quad n=\overline{1,m},\tag{5.1}$$

where $T^{(n)}(x,t)$ is the solution to (2.3) reduced to dimensionless form with the initial and boundary conditions of (2.6) and (2.9), while $W^{(n)}(x,t)$ satisfies the equation

$$\ddot{W}^{(n)}-\Delta W^{(n)}=F_3^{(n)}\tag{5.2}$$

with the initial and boundary conditions

$$W^{(n)}(\mathbf{x},t_0)=0,\quad \dot{W}^{(n)}(\mathbf{x},t_0)=0,\quad \frac{\partial}{\partial x_3}W^{(n)}(x_1,x_2,0,t)=0.$$

Equation (5.1) enables us to determine $W^{(n)}(x,t)$ unambiguously from the known functions $T^{(n)}(x,t)$ [4], which means that one can determine unambiguously $W^{(n)}(x_1,x_2,0,t)=\chi^{(n)}(x_1,x_2,t)$ for $(x_1^2+x_2^2)^{1/2}<r$, $r>0$ from the equation

$$\chi_3^{(n)}(x_1,x_2,t)-\Theta^{0(n)}(x_1,x_2,0,t)=\frac{1}{\sqrt{\pi t}}\int_0^\infty \exp(-\tau^2/4t)\,\chi^{(n)}(x_1,x_2,\tau)\,d\tau\tag{5.3}$$

Here we assume that the corresponding dimensionless variable substitution has been made in the functions $\chi_3^{(n)}$, $\Theta^{0(n)}$, but for convenience we retain the previous symbols.

The boundary-value problem for (5.2) is employed with the additional information derived from (5.3) and the auxiliary problem IV to determine $F_3^{(n)}(x)$, $n=\overline{1,m}$ uniquely.

6. We consider the solution to problem III. We represent the solution to

$$\ddot{v}^{(n)}-(\lambda^0+2\mu^0)\,\Delta v^{(n)}=-\beta^0\Delta T^{(n)}+F_1^{(n)}$$

as the sum of the homogeneous solution and two particular solutions corresponding to the two terms on the right in this equation $v^{(n)}=v^{0(n)}+v^{1(n)}=v^{2(n)}$; the function $v^{0(n)}=\mathrm{div}\,\mathbf{u}^{0(n)}$ can be taken as known. The function $v^{1(n)}(x,t)$ is found from the solution to

$$\ddot{v}^{1(n)} - (\lambda^0 + 2\mu^0)\Delta v^{1(n)} + \beta^0 \Delta T^{(n)} = 0, \quad n = \overline{1, m},$$
$$v^{1(n)}(\mathbf{x}, t_0) = 0, \ \dot{v}^{1(n)}(\mathbf{x}, t_0) = 0,$$
$$\frac{\partial}{\partial x_3} v^{1(n)}(x_1, x_2, 0, t) = 0,$$

where $T^{(n)}(\mathbf{x}, t)$ is uniquely determined from (2.3), (2.6), and (2.9) via the known function $F_3^{(n)}$. Consequently $v^{1(n)}(\mathbf{x}, t)$ can also be taken as known.

Now $F_1^{(n)}(\mathbf{x})$, $n = \overline{1, m}$ are found from

$$\ddot{v}^{2(n)} - (\lambda^0 + 2\mu^0)\Delta v^{2(n)} = F_1^{(n)}; \tag{6.1}$$
$$v^{2(n)}(\mathbf{x}, t_0) = 0, \ \dot{v}^{2(n)}(\mathbf{x}, t_0) = 0; \tag{6.2}$$
$$\frac{\partial}{\partial x_3} v^{2(n)}(x_1, x_2, 0, t) = 0 \tag{6.3}$$

by reference to the information

$$v^{2(n)}(x_1, x_2, 0, t) = \chi_1^{(n)}(x_1, x_2, t) - v^{0(n)}(x_1, x_2, 0, t) - v^{1(n)}(x_1, x_2, 0, t), \tag{6.4}$$
$$(x_1^2 + x_2^2)^{1/2} < r, \quad r > 0.$$

As system (6.1)-(6.4) splits up into m problems that are analogous to problem IV after reduction to dimensionless form, one can also take problem III as having been solved.

7. We now transfer to the direct determination of the characteristics for a weakly inhomogeneous weakly anisotropic thermoelastic medium. Problems I-III have enabled us to determine $F_1^{(n)}$, $F_2^{(n)}$, $n = \overline{1, m}$. If we assume that at the boundary of the half-space $x_3 \geq 0$ the characteristics of this medium and the derivatives of these with respect to x_3 coincide with the corresponding characteristics of a homogeneous isotropic medium, then $f^{(n)} = g^{(n)} = 0$ for $x_3 = 0$. This condition enables us to determine $f^{(n)}(\mathbf{x})$ uniquely from the equations $\operatorname{rot} f^{(n)} = F_2^{(n)}$, $\operatorname{div} f^{(n)} = F_1^{(n)}$. As $g^{(n)} = F_3^{(n)}$, in what follows we can take $f^{(n)}$, $g^{(n)}$, $n = \overline{1, m}$ as known.

The thermoelastic characteristics C_{ijkl}^{ε}, β_{ij}^{e}, K_{ij}^{ε} are found from two systems of linear differential equations containing first-order partial derivatives corresponding to (1.7) and (1.8). As the initial conditions of (1.9)-(1.11) have been chosen such as to satisfy homogeneous static thermoelastic equations with constant coefficients C_{ijkl}^0, β_{ij}^0, K_{ij}^0, we have $\dot{u}^{0(n)}(\mathbf{x}, t) \equiv 0$, $\dot{\Theta}^{0(n)}(\mathbf{x}, t) \equiv 0$, which means that $u_i^{0(n)} = u_i^{0(n)}(\mathbf{x}) = \varphi_i^{(n)}(\mathbf{x})$, $\Theta^{0(n)} = \Theta^{0(n)}(\mathbf{x}) = \psi^{(n)}(\mathbf{x})$,

$$\left(C_{ijkl}^{\varepsilon}\varphi_{k,l}^{(n)}\right)_{,j} - \left(\beta_{ij}^{\varepsilon}\psi^{(n)}\right)_{,j} = f_i^{(n)}; \tag{7.1}$$
$$\left(K_{ij}^{\varepsilon}\psi_{,i}^{(n)}\right)_{,j} = g^{(n)}, \quad i, j, k, l = \overline{1, 3}, \quad n = \overline{1, m}. \tag{7.2}$$

Here the superscript n, as previously, denotes that the quantity corresponds to test n, i.e., to boundary-value problem n.

System (7.1) contains 27 unknown functions $C_{ijkl}^{\varepsilon}(\mathbf{x}), \beta_{ij}^{\varepsilon}(\mathbf{x})$, while system (7.2) contains the six unknown functions $K_{ij}^{\varepsilon}(\mathbf{x})$. These systems can be simplified considerably by special choice of the functions $\varphi_i^{(n)}$, $\psi^{(n)}$, which in that case act as coefficients.

As natural boundary conditions for (7.1) and (7.2) one can take for example the conditions

$$C_{ijkl}^{\varepsilon}(x_1, x_2, 0) = 0, \quad i, j, k, l = \overline{1, 3}; \tag{7.3}$$
$$K_{ij}^{\varepsilon}(x_1, x_2, 0) = 0, \quad \beta_{ij}^{\varepsilon}(x_1, x_2, 0) = 0, \tag{7.4}$$

which corresponds to the assumption that the unknown thermoelastic characteristics coincide with the corresponding known characteristics of a homogeneous isotropic thermoelastic medium.

8. As an example we consider the diagnostic problem in the case where it is known a priori that the medium is slightly inhomogeneous but is isotropic. This assumption substantially reduces the number of unknown characteristics, of which only four remain: λ^{ε}, μ^{ε}, β^{ε}, K^{ε}, since for an isotropic thermoelastic medium

$$C^{\varepsilon}_{1111}=C^{\varepsilon}_{2222}=C^{\varepsilon}_{3333}=\lambda^{\varepsilon}+2\mu^{\varepsilon},\quad C^{\varepsilon}_{1212}=C^{\varepsilon}_{1313}=C^{\varepsilon}_{2323}=2\mu^{\varepsilon},$$
$$C^{\varepsilon}_{1112}=C^{\varepsilon}_{1113}=C^{\varepsilon}_{1123}=C^{\varepsilon}_{1223}=C^{\varepsilon}_{2213}=C^{\varepsilon}_{1323}=C^{\varepsilon}_{2212}=C^{\varepsilon}_{2223}=C^{\varepsilon}_{1213}=C^{\varepsilon}_{3313}=$$
$$=C^{\varepsilon}_{3312}=C^{\varepsilon}_{3323}=0,\quad C^{\varepsilon}_{1122}=C^{\varepsilon}_{1133}=C^{\varepsilon}_{2233}=\lambda^{\varepsilon},\quad \beta^{\varepsilon}_{ij}=\delta_{ij}\beta^{\varepsilon},\quad K^{\varepsilon}_{ij}=\delta_{ij}K^{\varepsilon}.$$

Then (7.1) and (7.2) take the form

$$[K^{\varepsilon}\psi^{(n)}_{,1}]_{,1}+[K^{\varepsilon}\psi^{(n)}_{,2}]_{,2}+[K^{\varepsilon}\psi^{(n)}_{,3}]_{,3}=g^{(n)},\tag{8.1}$$
$$n=\overline{1,m},\quad [(\lambda^{\varepsilon}+2\mu^{\varepsilon})\varphi^{(n)}_{1,1}+\lambda^{\varepsilon}(\varphi^{(n)}_{2,2}+\varphi^{(n)}_{3,3})]_{,1}+[\mu^{\varepsilon}(\varphi^{(n)}_{1,2}+\varphi^{(n)}_{2,1})]_{,2}+$$
$$+[\mu^{\varepsilon}(\varphi^{(n)}_{1,3}+\varphi^{(n)}_{3,1})]_{,3}-(\beta^{\varepsilon}\psi^{(n)})_{,1}=f^{(n)}_{1},\quad [\mu^{\varepsilon}(\varphi^{(n)}_{1,2}+\varphi^{(n)}_{2,1})]_{,1}+[(\lambda^{\varepsilon}+2\mu^{\varepsilon})\varphi^{(n)}_{2,2}+$$
$$+\lambda^{\varepsilon}(\varphi^{(n)}_{1,1}+\varphi^{(n)}_{3,3})]_{,2}+[\mu^{\varepsilon}(\varphi^{(n)}_{2,3}+\varphi^{(n)}_{3,2})]_{,3}-(\beta^{\varepsilon}\psi^{(n)})_{,2}=f^{(n)}_{2},$$
$$[\mu^{\varepsilon}(\varphi^{(n)}_{1,3}+\varphi^{(n)}_{3,1})]_{,1}+[\mu^{\varepsilon}(\varphi^{(n)}_{2,3}+\varphi^{(n)}_{3,2})]_{,2}+[(\lambda^{\varepsilon}+2\mu^{\varepsilon})\varphi^{(n)}_{3,3}+$$
$$+\lambda^{\varepsilon}(\varphi^{(n)}_{1,1}+\varphi^{(n)}_{2,2})]_{,3}-(\beta^{\varepsilon}\psi^{(n)})_{,3}=f^{(n)}_{3}.$$

We will determine only μ^{ε}. For this it is sufficient to perform a single test, i.e., $n = 1$, and this superscript will subsequently be omitted.

As initial conditions we take

$$\varphi_1=x_3,\ \varphi_2=0,\ \varphi_3=0,\ \psi=0.\tag{8.2}$$

It is readily seen that $\varphi_1, \varphi_2, \varphi_3, \psi$ thus selected satisfy the static equations of thermoelasticity for a medium with constant thermoelastic characteristics. We substitute (8.2) into (8.1) to get

$$\mu^{\varepsilon}_{,3}=f_1,\quad \mu^{\varepsilon}_{,1}=f_3,\quad 0=f_2,\quad 0=g.\tag{8.3}$$

As (8.1) has given four equations for the single function μ^{ε}, there are constraints imposed on f_1, f_2, f_3, and g implied by the very form of (8.3): $f_{1,1} = f_{3,3}$, $f_2 \equiv 0$, $g \equiv 0$. We use conditions (7.3), which in this case take the form $\mu^{\varepsilon}(x_1, x_2, 0) = 0$, to get

$$\mu^{\varepsilon}(x_1,x_2,x_3)=\int_0^{x_3}f_1(x_1,x_2,\eta)\,d\eta.$$

The physical significance of μ^{ε} requires that $\mu^{0} + \mu^{\varepsilon} > 0$.

LITERATURE CITED

1. V. G. Romanov, "Inverse problems in seismic and electromagnetic wave propagation," in: Methods of Solving Ill-Posed Problems and their Applications [in Russian], VTs Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1982).
2. W. Nowacki, Theory of Elasticity [Russian translation], Mir, Moscow (1975).
3. R. M. Garipov and V. B. Kardakov, "A Cauchy problem for the wave equation with a nonspatial initial manifold," Dokl. Akad. Nauk SSSR, 213, No. 5 (1973).
4. K. G. Reznitskaya, "The relationship between solutions to Cauchy problems for equations of different types and inverse problems," in: Mathematical Problems in Geophysics, Issue 5, Part 1 [in Russian], VTs Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1974).